From abstract mathematics to reality, and back again

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COMPUTEing things

Modelling: figure out the equations for a physical problem

Simulation: apply a numerical method to approximate the solution to the problem

Combined: analyse the physical problem, refine model, make predictions

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Numerical Analysis: analyse the numerical method

In Lund: Focus on differential equations $\frac{\mathrm{d}u(t,x)}{\mathrm{d}t} = f(t, x, u)$

My PhD research

Splitting schemes: a class of time-stepping methods

Idea:
Don't solve $\frac{\mathrm{d}u}{\mathrm{d}t} = f(u) + g(u)$

Instead, solve $\frac{\mathrm{d}u}{\mathrm{d}t} = f(u)$ and $\frac{\mathrm{d}u}{\mathrm{d}t} = g(u)$ separately, then cleverly combine

Convergence analysis:
Show error $= Ch^p$ as time step $h \to 0$

In words: more work = predictably better approximation

General problem setting

Nonlinear diffusion problems

Such as $\frac{\mathrm{d}u}{\mathrm{d}t} = \Delta(|u|^r)$ or $\frac{\mathrm{d}u}{\mathrm{d}t} = \nabla \cdot (|\nabla u|^{p-2}\nabla u)$

Add perturbation to split away:

E.g. $\frac{\mathrm{d}u}{\mathrm{d}t} = \Delta(|u|^r) + g(u)$

Abstract, but many different applications:

- Population dynamics
- Chemical reactions
- Time-delayed such systems

But who cares about the application?

Convergence analysis difficulties:

- Solutions have low regularity
  (cf. linear diffusion $\Rightarrow$ very regular)
- Cannot use linearity
- Often need to work in less nice spaces to be well defined, hard to visualize

Nevertheless! General approach shows strong (but low) convergence orders for many different splitting schemes under non-restrictive assumptions

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However... No one much interested; $<$ 25 combined citations on three first papers. Too narrow, not very applicable.

“Reality”: Differential Riccati Equations (DRE)

Towards end of PhD: Found that splitting schemes are well suited to an application from optimal control, differential Riccati equations:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = A^*P + PA + C^*C - PBB^*P$$

Operator-valued: fits into general abstract framework (complicated)

Matrix-valued: more concrete, applicable, lack of good large-scale methods

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First DRE paper now > 30 citations, more interested people

Postdoc 1: Chalmers and Gothenburg University

2 years as a postdoc at *Chalmers* and *Gothenburg University* with Axel Målqvist

Worked on FEM... i.e spatial discretizations, combined with non-splitting time stepping methods

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...and more DRE; higher order methods, adaptivity etc. So many possibilities with more regularity!

Postdoc 2: Max Planck Institute Magdeburg

2 years as a postdoc at the *Max Planck Institute for Dynamics of Complex Technical Systems* in Magdeburg with Peter Benner and Jens Saak

Even more DRE, but now more abstract again

- Analysis of operator-valued DRE solution structure (no numerical method!)
- Convergence analysis of related stochastic optimal control problems

But also: serious implementation work

The present

Currently employed here in Lund at the *Centre for Mathematical Sciences* (assistant professor)

Very generously funded by WASP:
Wallenberg AI, Autonomous Systems and Software Program

Research on numerical methods for optimization problems arising in machine learning applications

Idea: Reformulate as ODE, apply non-novel techniques that are novel in this context, make everything more abstract

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Take advantage of COMPUTE interdisciplinary courses, but keep working on something specific within your own discipline

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But luck requires an opportunity: these can be created

Important to know people where you might want to work; advance notice of positions, being kept in mind, etc.

Take-aways II

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A Plan B is not necessary if Plan A succeeds

My approach has been to spend much time on few opportunities, rather than little time on many

The end

Good luck in your careers!